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Foreword

(This foreword is not a part of IEEE Std 376-1975, IEEE Standard for the Measurement of Impulse Strength and Impulse Bandwidth.)

For many purposes in radio interference and electromagnetic compatibility work, it has become convenient to measure broadband emission in terms of peak voltage or field strength, especially with the use of automatic spectrum scanning instrumentation. Because of the simplicity of the impulse generator, it is used frequently for calibration purposes. This standard provides basic information relating to the use of this device and interpretation of measurements made using instruments based on it.

The measurement of impulse strength and impulse bandwidth, while conceptually relatively simple, in practice have been found difficult to perform with high accuracy and reproducibility. Various methods have been evaluated in various laboratories, and are mentioned in this standard. After much consideration, two techniques are recommended for general use.

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CLAUSE PAGE

IEEE Standard for the Measurement of Impulse Strength and Impulse Bandwidth

1. Introduction

The importance of the impulse as a concept in engineering work has come about not only because of the extensive use of pulses in many types of communication systems, but also because many sources of interference or radio noise are “impulsive” in nature. By an impulsive source one generally means a source of electromagnetic energy which can be represented by a series of discrete disturbances of low duty cycle. Usually, it will have a relatively broad frequency spectrum. The importance of the impulse is related to the fact that where the duration of the pulse generated by a given source is sufficiently short[1] in comparison with the reciprocal of the center or tuned frequency of a “narrow-band” network responding to it, the waveform at the output of the network is of a very definite shape practically independent of the input waveform, and has a peak value proportional to its “impulse strength.” Because of these relations, an impulse generator is useful for calibrating the network response, and the networks themselves may be characterized in terms of their equivalent “impulse bandwidth.”

Because the impulse has a broad spectrum, a quantitative measure of the “spectrum amplitude” is also a useful quantity.

2. Definitions

impulse strength: The area under the amplitude-time relation for the impulse.

NOTE — This definition can be clarified with the aid of Fig 1. Let $A(t)$ be some function of time having a value other than zero only between the times t_1 and $t_1 + \delta$. Then let the area under the curve $A(t)$ be designated by σ:

$$\sigma = \int_{\infty}^{\infty} A(t)\,dt = \int_{t_1}^{t_1+\delta} A(t)\,dt$$

[1]For a rectangular pulse, the spectrum is flat within about 1 dB up to a frequency for which the pulse duration is equal to 1/4 cycle. See Fig A.2.

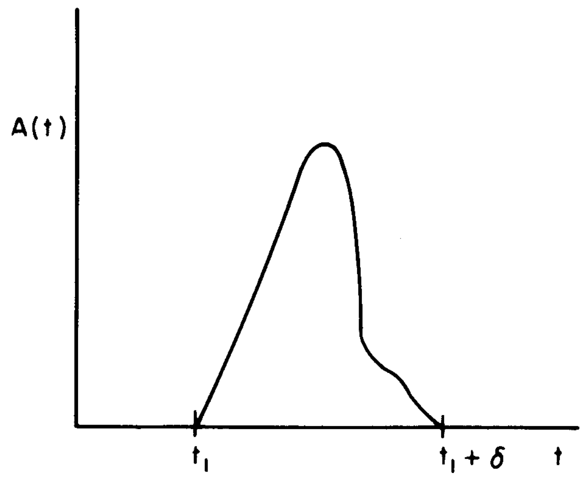


Figure 1— A Pulse of Arbitrary Shape

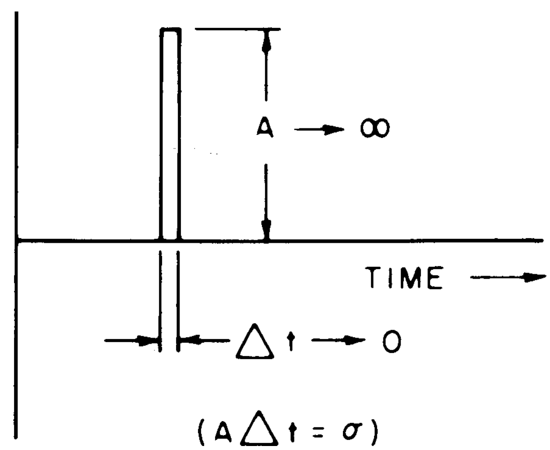


Figure 2— A Rectangular Pulse

To define the theoretical or ideal impulse, let *A(t)* vary in a reciprocal manner with δ such that the value σ remains constant, so that

$$\sigma = \lim_{\delta \to 0} \int_{t_1}^{t_1 + \delta} A(t)dt$$

In the limit the function $A(t)$ becomes an ideal "impulse" of "strength" σ. As an example, consider the function shown in Fig 2. Here a rectangular pulse of finite duration Δt and height A is shown. Now let $A = \sigma/\Delta t$ where σ is (for the present argument) an arbitrary constant, and let $\Delta t \to 0$. In the limit we have an impulse of strength σ. When $\sigma = 1$, one has a "unit impulse."

In many conventional applications the amplitude $A(t)$ has the dimension volts and σ then has the dimension volt-seconds.

spectrum intensity: (For spectra which have a continuous distribution of components (components are not discrete) over the frequency range of interest). The spectrum intensity is the ratio of the power contained in a given frequency range to the frequency range as the frequency range approaches zero. It has the dimensions watt-seconds or joules and is usually stated quantitatively in terms of watts per hertz.

spectrum amplitude: The voltage spectrum of a pulse can be expressed as[2]

[2]See IEEE Std 263-1965, Measurement of Radio Noise Generated by Motor Vehicles and Affecting Mobile Communications Receivers in the Frequency Range 25 to 1000 MHz.

$$V(\omega) = R(\omega) + jX(\omega) = \int_{-\infty}^{\infty} v(t)e^{-jwt}dt$$

where

$$R(\omega) = \int_{-\infty}^{\infty} v(t)\cos\omega t \ dt$$

$$X(\omega) = \int_{-\infty}^{\infty} v(t)\sin\omega t \ dt$$

and $\omega = 2\pi f$.

The spectrum then has the amplitude characteristic

$$A(\omega) = \sqrt{R^2(\omega) + X^2(\omega)} \qquad (\mathrm{V/rad})/\mathrm{s}$$

and the phase characteristic

$$\varphi(\omega) = \tan^{-1}\frac{X(\omega)}{R(\omega)}$$

The inverse transform can be written

$$v(t) = \frac{1}{\pi}\int_{0}^{\infty} A(\omega)\cos[\omega t + \varphi(\omega)]dw,$$

for real $v(t)$

The spectrum amplitude is also expressible in volts per hertz (volt-seconds) as follows:

$$S(f) = 2\mathrm{A}(\omega) \qquad (1)$$

It is this form that is used as the basis for calibration of commercially available impulse generators.

A practical impulse is a function of time duration short[1] compared with the reciprocals of all frequencies of interest. Its spectrum amplitude $S(f)$ is substantially uniform in this frequency range and is equal to twice the area under the impulse time function or 2σ. At frequencies higher than this it is still of interest to define the spectrum amplitude which will usually be less than 2σ.

In most broadband impulse generators adc voltage is used to charge a calibrated coaxial transmission line. The pulses are produced when the line is discharged into its terminating impedance through mechanically activated contacts. These mechanical contacts may be parts of either a vibrating diaphragm or mercury wetted relay switches. By proper choice of transmission line length and resistive termination, it is possible to produce impulses having a predictable uniform spectrum amplitude range.

The advent of solid-state switches has made it possible to switch on a sine wave for a precisely measurable time interval (τ), producing in the frequency band in the vicinity of the sine wave a spectrum simulating that produced by an impulse. The spectrum amplitude at that particular frequency can be measured in terms of a measurement of the amplitude of the sine wave when not switched, and a measurement of the on time (τ_0) for the switch.

impulse bandwidth: When an impulse is passed through a network with a restricted passband, the output generally consists of a wave train, the envelope of which builds up to a maximum value and then decays approximately. exponentially. The impulse bandwidth of such a network is defined as the ratio of that maximum value (when properly corrected for network sine wave gain at a stated reference frequency) to the spectrum amplitude of the pulse applied at the input. In networks with a single humped response, the reference frequency is taken as that at which the gain is maximum. (Overcoupled or stagger-tuned networks should not be used for measurement of spectrum amplitude of impulses.)

3. Theoretical Bases for Measurement

Two methods of measurement of spectrum amplitude and impulse bandwidth are described in detail in this standard. In addition, reference is made to other techniques which have been used from time to time.

The first method uses a video pulse technique. The second uses a substitution method in which the reference is a pulse-modulated sine wave generator whose parameters are measured.

Experience has shown that both of these techniques are capable of about equal accuracy. In any given laboratory one may be preferred to the other.

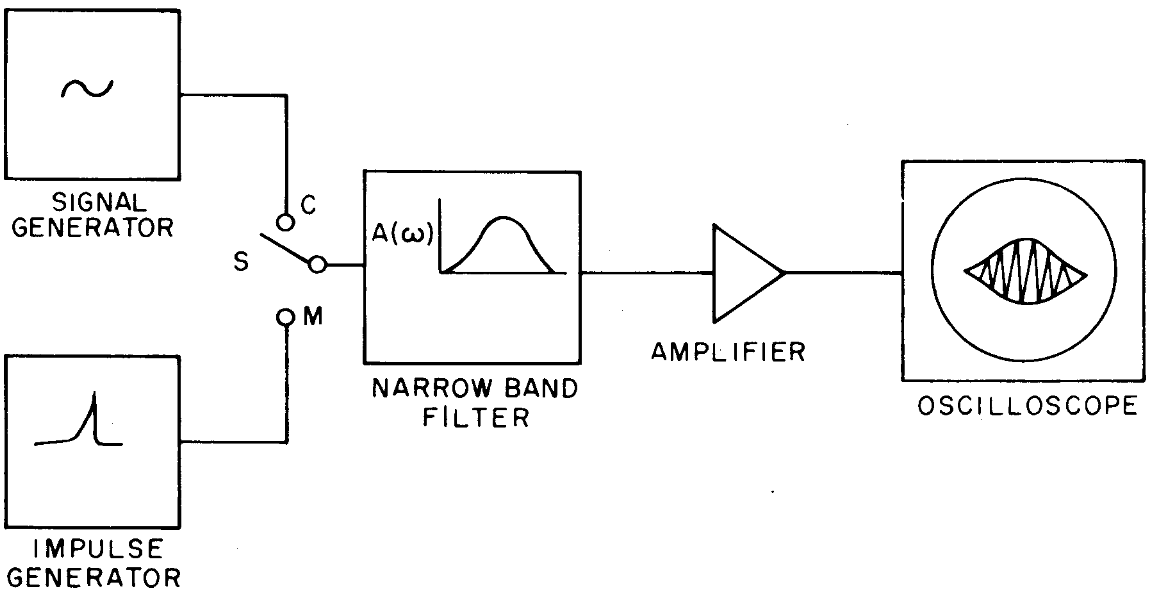


Figure 3— Conceptual Arrangement for Video Pulse Technique

3.1 Spectrum Amplitude: Video Pulse Technique

Fig 3 shows a representative arrangement. The narrow-band filter is assumed to be a linear network and must contain more than the equivalent of a single stage RLC network.[3] The amplifier is also assumed to be linear and to have a gain characteristic fiat with frequency within 1 dB over a frequency range $f_0 \pm \Delta F$ where ΔF is the 6 dB bandwidth of the narrow bandwidth filter. It should provide sufficient gain so that a suitable signal level is available at the oscilloscope to reduce errors caused by system noise to less than I percent. By means of Fourier analysis, one can show that the spectrum amplitude[4] is

$$S(f) = \frac{1}{G_0}\int_{-\infty}^{\infty} V(t)dt \qquad (2)$$

where $V(t)$ is the envelope of the voltage curve at the output of the amplifier and G_0 is the overall gain of the system to the same point at the frequency at which the response of the narrow-band filter is maximum. It should be noted that this relation has been proved only for the case of a narrow-band filter where the amplitude and phase characteristics of

[3] A single stage RLC network does not have a finite impulse bandwidth.
[4] GESELOWITZ, D. B., Response of Ideal Radio Noise Meter to Continuous Sine Wave, Recurrent Impulses, and Random Noise. *IRE Transactions on Radio Frequency Interference*, vol RFI-3, no 1. pp 2–10, May 1961.

the filter are symmetrical. It is believed that this can be sufficiently closely approximated in practice so as to be valid. Hence, if the filter passband is sufficiently small, by measuring the area under the curve representing $V(t)$ and by a sine wave gain measurement, the spectrum amplitude $S(f)$ can be measured.

For simple unidirectional pulse shapes, one generally expects the spectrum amplitude to decrease with frequency at frequencies above about one-half of the reciprocal of the pulse duration. To obtain the spectrum amplitude of the pulse, measurements are carried out with a series of filters having passbands at an appropriate number of frequencies throughout the frequency range of interest.

3.2 Spectrum Amplitude: Pulse-Modulated Sine Wave Technique

The method is based upon the spectrum of a regularly repeated rectangular pulse-modulated sine wave.[5] For such a wave the spectrum can be shown to consist of discrete components spaced f_0 in frequency, where f_0 is the pulse repetition frequency. When the amplitudes of the components versus frequency are plotted, their envelope has the well-known (sin x)/x shape, centered at the sine wave frequency f. Near this frequency the amplitude of each component is, as shown in the Appendix, $f_0 \tau_0 E$, where τ_0 is the duration of the rectangular pulse and E is the peak value of the sine wave. Then the “spectrum amplitude” can be defined as the “peak voltage per unit frequency,” or

$$S(f) = E\tau_0 \tag{3}$$

If such a spectrum is passed through a narrow-band filter having, however, a bandwidth sufficiently large to pass many (at least 10) frequency components, the response to each pulse train will be nearly identical to that of the “ideal” impulse discussed in Section 2.[6]

Thus, if a pulse-modulated sine wave generator is constructed for which τ_0 and E can be accurately determined, one can calibrate another pulse generator by comparing the effects produced by it and the standard generator in a narrow-band filter. With a proper circuit design, E can be measured directly and τ_0 can be measured by finding the first null in the (sin x)/x spectrum which is located at 2 π/τ from the frequency f.

3.3 Impulse Bandwidth

From the definition of impulse bandwidth we have the formula:

$$\Delta F_{imp} = \frac{V(t)_{max}}{G_0 S(f)} \tag{4}$$

where $V(t)_{max}$ is the maximum value of the voltage response of the network to an applied impulse voltage. Hence, a measurement of this maximum response, together with the previously measured spectrum amplitude and network gain, will give the impulse bandwidth.

Using Eq 3 in Eq 4 we have

$$\Delta F_{imp} = \frac{V_{t\ max}}{G_0 E \tau_0} \tag{5}$$

[5]This technique was originally described in the Society of Automotive Engineers Aerospace Recommended Practice ARP 1267, EMI Measurement Impulse Generators, Standard Calibration Requirements and Techniques.

[6]PALLADINO, J. R., A New Method for the Spectral Density Calibration of Impulse Generators, *IEEE Transactions on Electromagnetic Compatibility*, vol EMC-13, p 2, Feb 1971.

Thus with the pulse-modulated sine wave technique the impulse bandwidth can be obtained from the ratio of the equivalent input level of the maximum of the pulse envelope response to the sine wave level multiplied by the reciprocal of the pulse length.

3.4 Use of RMS Values

It is customary to calibrate radio-frequency devices in terms of the rms value of a sine wave, since all signal generators are calibrated that way. Hence for practical purposes it is convenient to follow that practice in dealing with impulse phenomena. Thus it is conventional to calibrate impulse generators in terms of their “rms equivalent” spectral levels. With this convention, Eq 1 is written

$$S(f)_{\mathrm{rms}} = \sqrt{2}A(\omega) \tag{1a}$$

Eq 2 becomes

$$S(f)_{\mathrm{rms}} = \frac{1}{G_0}\int_{-\infty}^{\infty} V(t)_{\mathrm{rms}}\,dt \tag{2a}$$

where the envelope amplitude $V(t)_{\mathrm{rms}}$ is now expressed in terms of equivalent rms sine wave levels. Eq 3 becomes

$$S(f)_{\mathrm{rms}} = E_{\mathrm{rms}}\tau_0 \tag{3a}$$

Eq 4 becomes

$$\Delta F_{\mathrm{imp}} = \frac{V(t)_{\mathrm{rms\,max}}}{G_0 S(f)_{\mathrm{rms}}} \tag{4a}$$

and Eq 5 becomes

$$\Delta F_{\mathrm{imp}} = \frac{V_{t\,\mathrm{rms\,max}}}{G_0 E_{\mathrm{rms}}\tau_0} \tag{5a}$$

Eqs <$attribute., <$attribute., <$attribute., <$attribute., and <$attribute. are referenced in the measurement procedure described below.

4. Measurement Procedures

The arrangement of test equipment for use of both methods is shown in Fig 4. Note that the narrow-band filter of Fig 3 has been replaced by a communications type receiver or a radio noise meter or similar device. This receiver is tuned to the frequency f_1 at which one wants to calibrate the spectrum amplitude of the pulse generator. As noted previously it should not have stagger tuned or overcoupled filters. The switches and interconnecting cables should be of the matched coaxial type for frequencies above about 1 MHz.

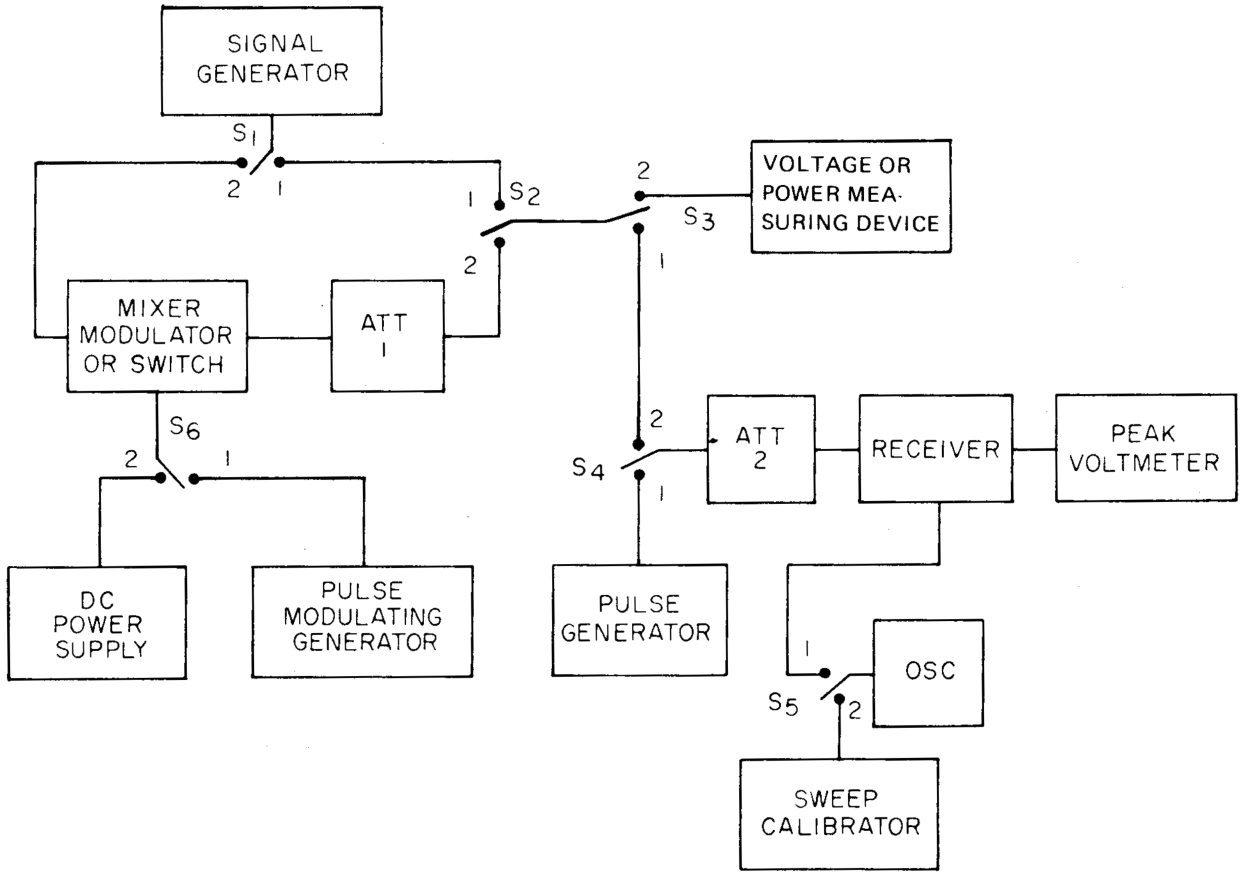


Figure 4— Arrangement for Spectrum Amplitude Measurement

4.1 Video Pulse Technique

For this procedure the mixer, attenuator 1, dc power supply, pulse-modulating generator, and the peak voltmeter are not needed. Certain precautions should be observed as follows:

1) The impulse generator and signal generator should be matched to the receiver input impedance as prescribed for its normal use, and the oscilloscope (or amplifier, if used) connection should be made at an appropriate place on the receiver output so as to provide conditions simulating normal operations. If the filter input impedance match is not within a VSWR of 1.2 over the passband of the receiver, the resistive attenuator 2 shown should be adjusted for at least 10 dB of attenuation.
2) The amplifier and the oscilloscope should have flat frequency characteristics over the passband of the receiver and linearity of response should be maintained throughout the system.
3) Frequently, the receiver or radio noise meter will utilize automatic gain control. The measuring point in such a case is usually located between the last IF stage and the detector. The detector should be removed in order to avoid distortion of the envelope due to the detection process and to disable the automatic gain control circuit which might otherwise distort the response envelope. A check should be made to determine if the amplifier has been detuned as a result. If so, the final IF amplifier should be retuned.
4) The oscilloscope should have a linear deflection system in both horizontal and vertical directions.
5) Since the receiver contains active elements, it should be determined that the impulse generator is not causing the active elements to be driven into nonlinear regions.

The calibration procedure is as follows:

1) Initially all switches are placed in the position 1. The pulse generator output, the input attenuator, the amplifier gain, and oscilloscope controls should be adjusted to provide a stable undistorted pulse response envelope on the oscilloscope screen. with minimum observable internal amplifier noise. To insure freedom from pulse overload effects the input attenuator should be changed and the output checked for linearity.
2) The area (A) (in square centimeters) of the pulse response envelope (positive and negative sections properly summed) is then obtained by photographing the oscilloscope display, or by tracing the pattern on graph paper, and then using a planimeter.
3) The vertical axis is calibrated in equivalent input rms microvolts per centimeter by placing switch S_4 in the position 2 and adjusting the signal generator frequency for maximum response and output to obtain a convenient peak-to-peak response of d_1 centimeters for an rms voltage of V microvolts at the signal generator terminals. Note that for some signal generators a correction factor, which is dependent on the load impedance, may be necessary to convert indicated output voltages to terminal output voltages. The signal generator may be directly calibrated by placing switch S_3 in position 2. Again it is necessary that the input impedance of the calibrating device be identical to that of attenuator 2.
4) The time axis of the oscilloscope is calibrated in microseconds per centimeter (t_1) by substituting an appropriate accurately calibrated oscillator at the vertical input terminals by means of switch S_5.
5) The spectrum amplitude in microvolts per megahertz is then given by the formula:

$$S_{\text{rms}}(f) = \frac{At_1V}{d_1}$$

6) The impulse bandwidth in megahertz is obtained by the formula:

$$\Delta F_{\text{imp}} = \frac{V}{S_{\text{rms}}(f)}\frac{d_2}{d_1} = \frac{d_2}{At_1}$$

where d_2 centimeters is the maximum vertical dimension measured on the area A.

7) The measurement should be repeated for each tuned receiver frequency of interest f_2, f_3, …

4.2 Pulse-Modulated Sine Wave Technique

In this technique the oscilloscope and sweep calibrator are not needed. The key component is that identified as a “mixer, modulator, or switch.” It must have the ability to pass the signal generator output when supplied from the pulse-modulating generator at the same amplitude, when the pulse generator is “on,” as when it is supplied from the dc power source, and to totally block the signal generator output when the pulse-modulating generator is “off.” The waveform of the output of the pulse-modulating generator should be sharply rectangular so there is no substantial ambiguity in the “on” time τ_0.

The measurement procedure is as follows:

1) Switch 1 is placed in position 1, the receiver gain (or attenuator 3) is adjusted for a convenient reference indication on the peak voltmeter.
2) With switches S_1, S_2, S_4, and S_6 in position 2 and switch S_3 in position 1, the frequency of the signal generator is adjusted for maximum output on the peak voltmeter.
3) With switch S_6 in position 1, the signal generator output is adjusted to obtain the same reference indication on the voltmeter previously obtained in step (1).
4) Switches S_3 and S_6 are both now placed in position 2 and the power level noted.
5) The spectrum amplitude output of the “item to be calibrated” is calculated from the following equation (in practice, the meter readout of the power measuring device is calibrated in terms of spectrum amplitude):

$$S_{\text{rms}} = \tau_0\sqrt{\frac{P|Z|}{\cos\theta}}$$

where

P = power in watts

$/Z/$ = absolute impedance of the power measuring device in ohms

θ = phase angle between the resistive component and the impedance $/Z/$, generally assumed to be equal to 0°,

τ_0 = pulsewidth of the pulse signal used to modulate the cw signal, in seconds

6) To obtain the impulse bandwidth one follows the above procedure through step (3). Then with switch S_6 in position 2, attenuator 1 is adjusted for reference indication on the peak voltmeter. Let the attenuation inserted be U dB.
7) The impulse bandwidth in hertz is given by Eq 5a or

$$\Delta F_{\text{imp}} = \frac{1}{\tau_0}\log^{-1}\frac{U}{20}$$

8) The measurement should be repeated for each tuned receiver frequency of interest $f_2, f_3, \ldots$

5. Alternative Measurements of Impulse Strength

NOTE — In addition to the aforesaid, other methods have been used in impulse calibration as described in the following sections. For particular purposes or in particular laboratories they may produce good accuracy but they are not described in detail here, nor are they recommended for general use. Those interested should consult appropriate references.

5.1 Standard Transmission Line Method[7]

A transmission line of length corresponding to a propagation time τ and charged to a voltage V_0 is discharged into a load resistance equal to the characteristic impedance of the line. The transmission line is here considered to consist of the actual line as well as the charged section of the line contained in the switch housing. As is shown in the reference in footnote 7, the spectrum amplitude $S(f)$ has the value $2V_0\tau$ (or $\sqrt{2}V_0\tau$ in terms of rms values) in the low-frequency portion of the spectrum of the resulting pulse in which the spectrum amplitude is constant with frequency, this amplitude being independent of the existence of certain stray impedances between the line and the load resistor (for example, inductance or resistance) or of finite switching time. The value $S(f)$ may be calculated directly from the measured values of V_0 and τ in the frequency range in which the measured spectrum amplitude has been found to be constant.

5.2 Harmonic Measurement[7]

This method may be used for pulse generators producing a sequence of pulses with sufficiently high and stable repetition frequency. In this case one has a discrete rather than a continuous spectrum. When the pulse repetition frequency F exceeds the value of the bandwidth of the measuring receiver the latter may select one line from the pulse spectrum. In this case the spectrum amplitude (which has significance only for bandwidths large compared with F) may be determined as follows:

$$\varphi(f, k) = \frac{V_k}{F}$$

where V_k is the rms amplitude of the k^{th} harmonic. In the event that the repetition rate is not sufficiently high so as to prevent more than one spectrum component from contributing to the output. the effects of the additional components may be taken into consideration if the bandpass characteristics of the filter are sufficiently well known.[6]

[7]Material is largely excerpted from the determination of the amplitude relationship specified in CISPR Publications 1, 2, and 4, Report no 42, Oct 1970.

The pulse generator may then be used to calibrate the pulse response characteristics of a network whose bandwidth is sufficiently wide to accept many harmonic components (approximately 10 or more within the 6 dB bandwidth).

5.3 Energy Method

Since for applied impulses the spectra obtained can be expected to vary slowly with frequency in a manner which can be reasonably accurately predicted, it is possible to make a measurement of spectrum intensity and to calculate the spectrum amplitude from it. This requires a power measurement.[8] In this technique the effective power bandwidth of a bandpass filter is measured. The techniques for doing this by measuring the sine wave response of the filter are well known. The power bandwidth is then

$$\Delta f_p = \frac{1}{|G(f_0)|^2}\int_0^\infty |G(f)|^2 df$$

where $G(f)$ is the voltage gain at any frequency f, and f_0 is the gain at the reference frequency (usually the frequency for maximum filter response).

The spectrum amplitude is then given by

$$S_{\text{rms}} = \sqrt{2}\sigma = \frac{1}{G(f_0)}\sqrt{\frac{\overline{P}R_0}{f_r \Delta f_p}}$$

where

$\overline{P}$ = equivalent average power at the input
R_0 = input resistance in ohms
f_r = (uniform) repetition rate of applied impulses, in hertz
Δf_p = power bandwidth

This technique has the advantage of enabling the measurement to be made in terms of some relatively accurately measurable quantities, namely average power and pulse repetition rate.

Its accuracy is limited by the following:

1) The input impedance of the measuring network should be a pure known resistance at the frequency of measurement.
2) The power measuring equipment must be able to respond accurately to waveforms having a high peak to average ratio.
3) Interpulse background circuit noise may contribute substantially to the pulse noise unless a suitable blanking technique is used.
4) The impulse source may not be capable of operating at a sufficiently high repetition rate or its output level may be a function of frequency, At low repetition rates, items (2) and (3) may present serious accuracy limitations.

[8]ANDREWS, R. B., Jr, An Impulse Spectral Intensity Measurement System, *IEEE Transactions on Instrumentation and Measurement*, vol IM-15, no 4, Dec 1966.

Annex Pulse-Modulated Carrier Analysis

(Informative)

Consider a rectangular pulse train as shown in Fig 1. From the exponential form of the Fourier series:

$$C_n = \frac{1}{T}\int_{\tau_0/2}^{\tau_0/2} E e^{-jn\omega_0 t}\,dt \tag{A-1}$$

$$C_n = \frac{E\tau_0}{T}\frac{\sin\left(n\omega_0\frac{\tau_0}{2}\right)}{n\omega_0\frac{\tau_0}{2}} \tag{A-2}$$

which is of the well-known form (sin X)/X, where $X = n\omega_0\tau_0/2$. The frequency spectrum is shown in Fig 2.

Next consider a cw signal, modulated by a rectangular pulse train as shown in Fig 3. Note that in this development, a time synchronism has been assumed between the rf carrier frequency wave and the pulse-modulation frequency wave. This has been done to simplify the mathematics. Actually, if there are at least 10 cycles of rf carrier within each pulse, the theoretical error introduced by this assumption can be shown to be negligible.

$$f(t) = \begin{cases} E\cos\omega_c t & -\frac{\tau_0}{2} < t < \frac{\tau_0}{2} \\ 0 & \text{elsewhere} \end{cases}$$

and ω_c = 2DCL103/T.

The Fourier coefficient may be represented as:

$$C_n = \frac{1}{T}\int_{-\tau_0/2}^{\tau_0/2} f(t) e^{-jn\omega_0 t}\,dt \tag{A-3}$$

This may be written in the form:

$$C_n = \frac{E\tau_0}{2T}\left(\frac{\sin(n\omega_0-\omega_c)\frac{\tau_0}{2}}{(n\omega_0-\omega_c)\frac{\tau_0}{2}} + \frac{\sin(n\omega_0+\omega_c)\frac{\tau_0}{2}}{(n\omega_0+\omega_c)\frac{\tau_0}{2}}\right) \tag{A-4}$$

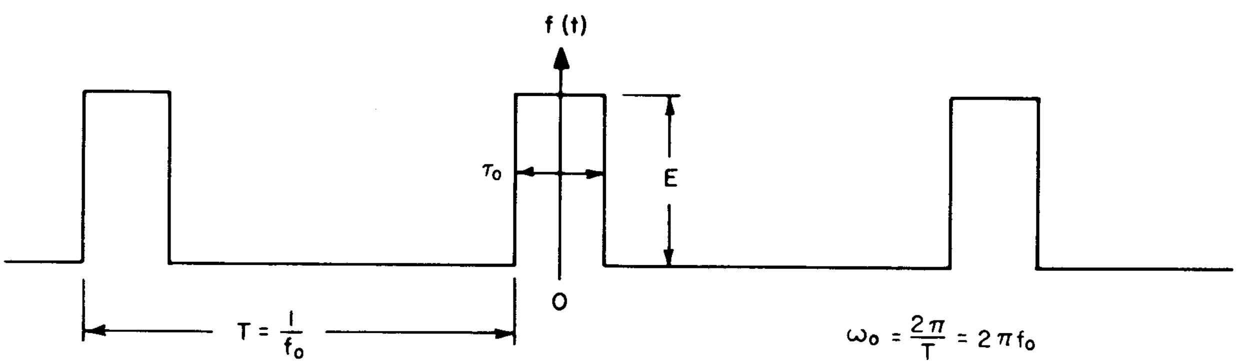


Figure A.1—Rectangular Pulse Train

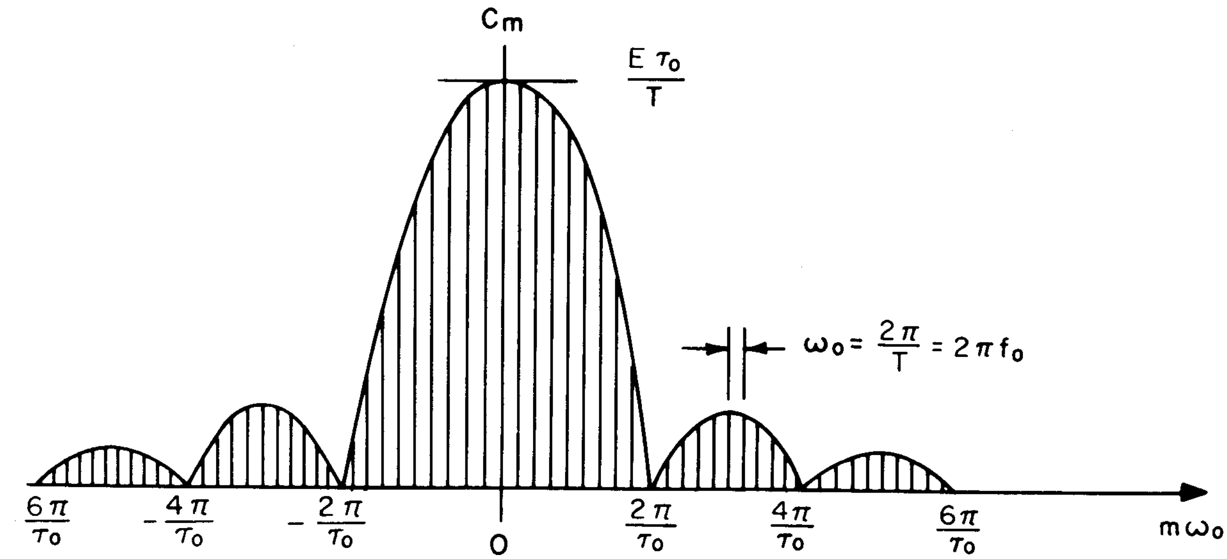


Figure A.2—Frequency Spectrum of the Rectangular Pulse Train

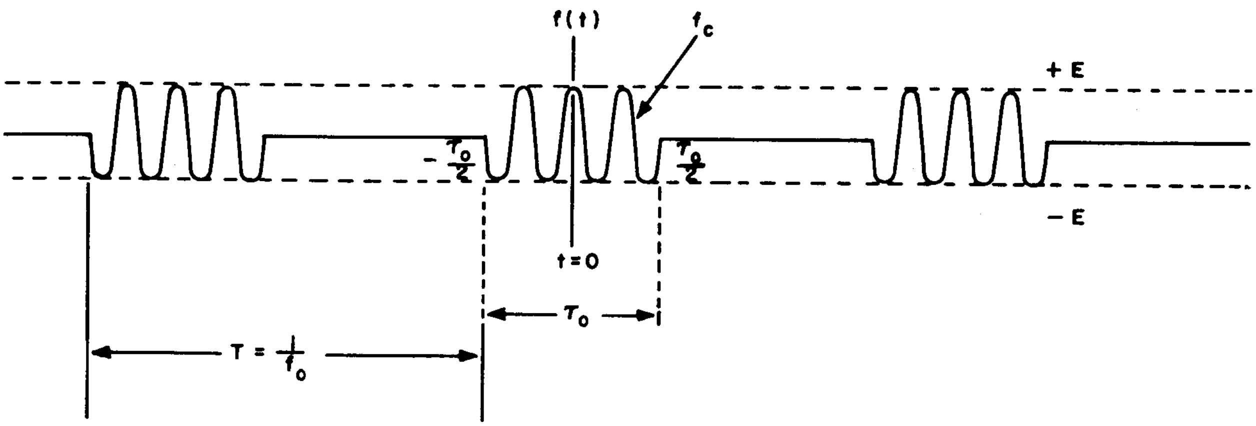


Figure A.3—CW Signal Modulated by a Rectangular Pulse Train

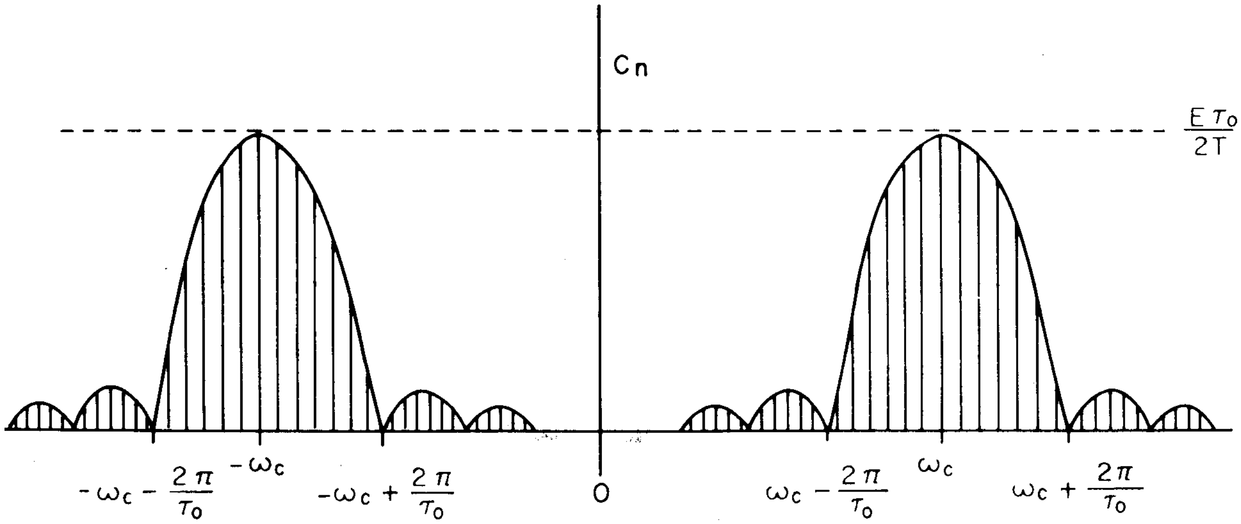


Figure A.4—Frequency Spectrum of the Pulsed CW Signal

The frequency spectrum of the pulsed cw signal is shown in Fig 4.

Eq A.2 is similar to Eq A.4 except for the symmetrical translation of the original frequency components $n\omega_0$ to both sides of the origin, namely $(n\omega_0 - \omega_c)$ and $(n\omega_0 + \omega_c)$. The relationship between the frequency spectrum of a rectangular pulse and the pulsed cw is shown in Fig A.5.

It is apparent that the frequency spectrum of a rectangular pulse main lobe is centered about zero. However, the pulsed cw signal is centered about ω_0. That is, the spectrum is translated from zero to + ω_c and zero to - ω_c. As a result, there are n spectral lines from 0 to ω_c and n spectral lines from 0 to - ω_c, where the spectral lines are separated by the repetition frequency f_0 of the rectangular pulse.

To evaluate Eq A.4, let $\omega_c = 0$. This results in an equation identical to Eq 2, the rectangular pulse train, thus:

$$C_n = \frac{E\tau_0}{2T}\frac{\sin(n\omega_0 - \omega_c)\frac{\tau_0}{2}}{(n\omega_0 - \omega_c)\frac{\tau_0}{2}} + \frac{\sin(n\omega_0 + \omega_c)\frac{\tau_0}{2}}{(n\omega_0 + \omega_c)\frac{\tau_0}{2}} \quad \text{(A-5)}$$

$$C_n = \frac{E\tau_0}{T}\frac{\sin(2\pi n f_0)\frac{\tau_0}{2}}{(2\pi n f_0)\frac{\tau_0}{2}} \quad \text{(A-6)}$$

The maximum amplitude occurring at the center of the spectrum $C\ n$ is

$$C_n = f_0\tau_0 E \quad \text{(A-7)}$$

It can also be shown that the minima or nulls will occur when

$$\frac{2\pi\tau_0}{T} = 2\pi N$$

where $N = 1, 2, 3, 4, \ldots$

The maxima or peaks will occur halfway between the nulls at:

$$n = \frac{(2N-1)}{2}\left(\frac{T}{\tau_0}\right)$$

It is readily apparent that the first null in the spectrum will occur at a frequency of $1/\tau_0$ in respect to the center of the spectrum f_0. Those spectral lines close to the center of the spectrum will have relatively the same amplitude when the bandwidth of the receiver or indicator is narrow relative to the pulse signal spectrum. For small angles (Eq A.6) the sine is very nearly equal to the angle in radians; hence, there is negligible drop-off until the angle exceeds about 0.04 rad and the $(\sin x)/x$ term is nearly equal to unity.

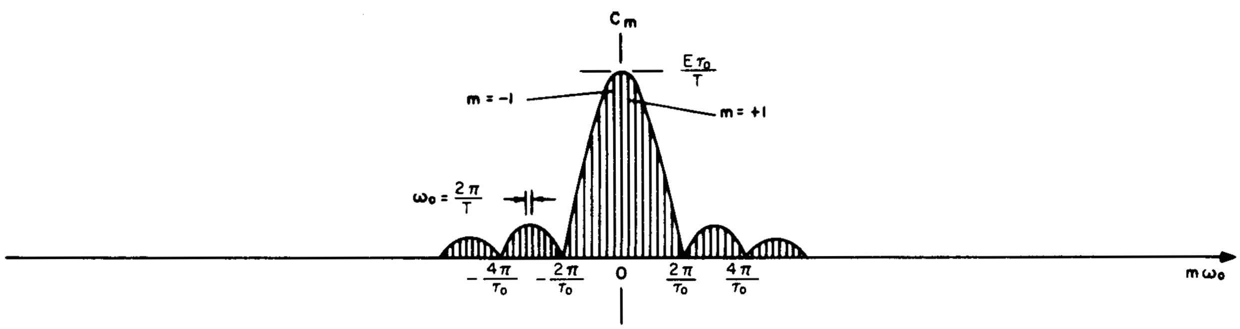
Evaluation of Eq A.6 for all spectral lines within ±5 percent (± 5 percent of $1/\tau_0$) of the spectrum center shows the roll-off in amplitude will not exceed 0.4 percent or 0.04 dB.

Thus, the spectrum amplitude of an impulse generator can be accurately calibrated with negligible error.

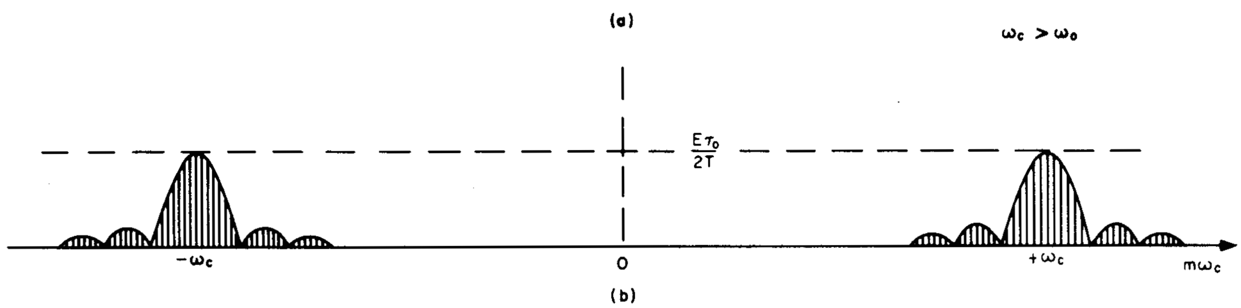
The amplitude of the spectrum center is expressed in Eq A.7 as the product of the pulse repetition frequency f_0, the pulsewidth τ_0, and the amplitude E of the cw signal. It can be shown that the spectrum amplitude of the spectrum center is independent of f_0, the pulse repetition frequency.

This may be illustrated considering a case in which the f_0 is doubled. The amplitude of the spectrum center would be doubled, but the corresponding number of spectral lines per given unit of bandwidth would be halved. The spectrum amplitude would remain constant since it is an expression of voltage per unit of bandwidth.

From Eq A.7, $C\,n = f_0\,\tau_0\,E$. Divide this by f_0 to get $S = \tau_0\,E$, where S is the spectrum amplitude.



(A) Frequency Spectrum of a Rectangular Pulse



(B) Frequency Spectrum of a Pulsed CW Signal

Fig A5